Forces on a blues harp reed

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Abstract

Blues harp reeds are free reeds. They can move freely due to the slots in the reed plate and oscillate almost sinusoidally in the first transverse mode of a clamped beam [Mi 99].

The motion of a reed tip (and thus reed motion) can be modeled by an abstract 1-point oscillator equation, where effective values have to be used for some parameters. This linear equation can be used to compare the influence of different forces on reed motion.

The oscillation is generated and kept going by interaction with the fluctuating air pressure in the channel. Here, the amplitude of the oscillatory component of the pressure force on the reed in the oscillator equation is an order of magnitude smaller than the amplitude of the elastic restoring force. The influence of the pressure force is explained in frequency domain by a slight phase shift of the fundamental oscillations of reed and pressure, which provides for the necessary energy supply and for a playing frequency deviating from the resonant frequency of the reed. Apart from that, a blues harp reed oscillates "almost by itself" due to the interplay of elastic restoring force and mass inertia. This is ultimately due to the fact that the reed oscillates unhindered and weakly damped.

In a supplementary section, it is argued why it is better not to call the pressure force acting on the reed a "Bernoulli force", even if it is related to a Bernoulli equation.

1 Comparison of forces

1.1 Oscillator equation and effective quantities

The reed of a blues harp is physically a beam clamped at one end. Stroboscope recordings and spectral analyses show that the beam oscillates in the first transverse mode. The reed movement can therefore be described by the deflection x of a point at the tip of the reed that is centered with respect to the two reed surfaces.

Furthermore, a blues harp reed oscillates almost sinusoidally [Mi 99], so that its motion can be described in a good approximation by a harmonic oscillator equation for the displacement x:

$$m\ddot{x} + 2rm\dot{x} + kx = S \cdot p \tag{1}$$

It is assumed that a homogeneous pressure p acts in the interior of the chamber across the surface of the reed and that the pressure on the surface facing outward can be neglected. Since (1) is intended to describe in particular the decay behavior of the reed, the calculated and measured relaxation time τ must agree. Thus, the damping constant $r = 1/\tau$ in the oscillator equation agrees with the damping constant measured on the reed. In contrast, m , k and S denote effective quantities [Mi 99, Appendix A] [MCV 01].

By pulling at the tip of the reed and measuring the corresponding deflection, a spring constant k_S can be determined. This results in an effective spring constant k, which is used to calculate the restoring force $F = -kx$ in the oscillator equation:

$$k = \frac{16}{15} k_S \tag{2}$$

$$F = -k \cdot x \tag{3}$$

The homogeneous pressure difference p between both surfaces of the reed causes a pressure force F, which occurs on the right side of (1) as an external force and which is the product of p with an effective area S. If L and B are the length and the width of the oscillating surface, then:

$$S = \frac{2}{5} L \cdot B \tag{4}$$

$$F = S \cdot p \tag{5}$$

With the abbreviation:

$$\omega_0^2 = \frac{k}{m} \tag{6}$$

the equation of motion (1) can be transformed as usual:

$$\ddot{x} + 2r\dot{x} + \omega_0^2 x = \frac{S \cdot p}{m} \tag{7}$$

Here ω_0 turns out to be the resonant frequency of the abstract 1-point oscillator described by (7), and this should also hold for the actual harp reed modeled by this equation. Due to the weak damping of an oscillating blues harp reed, its resonant (circular) frequency ω_0 is approximately equal to its eigenfrequency, which is measurable as the pseudo eigenfrequency of the decaying oscillation after the reed is plucked. After measuring ω_0 and k, the quantity m is determined by (6):

$$m = \frac{k}{\omega_0^2} \tag{8}$$

The mass m of the equivalent oscillator is therefore also an effective quantity. For example, the effective mass of a reed in channel #4 of a C harp is only about 10% of the actual oscillating reed mass.

1.2 Force comparison using the oscillator equation

The deflection x in the oscillator equation (1) can be identified with the deflection of the reed tip. How much the different forces acting on the reed influence its motion can consequently be read from the abstract 1-point equation:

$$m\ddot{x} = -2rm\dot{x} - kx + S \cdot p \tag{9}$$

It should be noted that mass m, spring constant k and area S are effective quantities.

In particular, the oscillator equation can be used to compare the influence of pressure force and elastic spring force on the reed motion, where the pressure force comes from the pressure fluctuations in the reed chamber.

1.3 Force comparison in the time domain

Laurent Millot measured reed movement and pressure difference at the reeds [Mi 99]. In the following we refer to his figures [Mi 99, p. 56, Fig. 21, 22] rsp. [Mi 99, p. 59, Fig. 26, 27], which show the deflection $x\,(t)$ and the pressure difference $p\,(t)$ at the reed for a normal draw note and for a semitone bend on channel #4 of a G-Harp, respectively.

With length and width of the reed [Mi 99, p. 37] an effective area $S = 1.1 \cdot 10^{-5} m^2$ results, the effective spring stiffness is (rounded) $k = 50N/m$ according to [Mi 99, p. 44]. For a normal draw note, the amplitude of oscillation is about $x = 1.5mm$, the pressure varies (roughly estimated) with $p = 150Pa$ about its time average value. For the draw bend, the corresponding quantities are $x = 0.75mm$ resp. $p = 400Pa$.

For a normal draw note, the restoring force consequently oscillates, according to (3), with an amplitude of $75mN$, while the pressure force on the reed fluctuates with $1.7mN$. The oscillating part of the restoring force is thus by a factor of 45 stronger than the pressure fluctuations.

For a draw bend, the restoring force with an amplitude of $38mN$ fluctuates by a factor of 9 more than the pressure force with an amplitude of $4.4mN$.

1.4 Force comparison in frequency domain

1.4.1 Sinusoidal oscillations with playing frequency

For sinusoidal pressure fluctuations $p = \hat{p}\cos\omega t$, the oscillator equation (7) has the stationary solution $x = \hat{x}\cos(\omega t + \varphi)$ with

$$\hat{x} = \frac{S \cdot \hat{p}}{m} \cdot \frac{1}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left(2r\omega\right)^2}} \tag{10}$$

$$\tan\varphi = \frac{2r\omega}{\omega^2 - \omega_0^2} \tag{11}$$

In the steady state region, reed oscillations and air pressure oscillations are periodic with the same period and thus also the same frequency ω, so that (10) and (11) can be applied with ω as fundamental and playing frequency. Realistic pressure fluctuations can be written as a Fourier series in ω because of the periodicity, and the harmonics $n\omega (n > 1)$ have comparatively little effect on the shape of the reed oscillation because of the low damping. In fact, the reed oscillation turns out to be nearly sinusoidal [BAB 98, Mi 99].

1.4.2 Restoring force and pressure force for a reed D in channel #4 of a C-Harp

In what follows, the reed D in channel #4 of a C harp will be exemplarily considered. Its eigenfrequency measured by plucking is $f_0 = 597Hz$, the effective spring stiffness equals $42N/m$, the effective mass is $3.0mg$, the damping constant is $r = 7s^{-1}$.

We want to compare restoring force $F_{elastic} = -k \cdot x$ of the reed and compressive force $F_{pressure} = p \cdot S$. The following applies to the ratio of the amplitudes using (10):

$$\frac{F_{elastisch}}{F_{Druck}} = \frac{k}{m} \cdot \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2r\omega)^2}} \qquad (12)$$

- For a normal draw note with $f = 586Hz$, (12) shows that the elastic restoring force of the reed is 27 times stronger than the pressure force (the "Bernoulli force") on the reed. The phase shift φ from (11) equals $-5.7°$, so the reed oscillation lags slightly behind the pressure oscillations. Since the restoring force is directed opposite to the displacement, the pressure force and restoring force oscillate approximately in opposite phase.

- For a semitone bend with $f = 555Hz$ the restoring force on the draw reed is 7 times stronger than the pressure force. The phase shift between reed movement and pressure is $-1.5°$. Pressure force and restoring force oscillate almost in opposite phase.

1.5 Blues harp reeds move almost by themselves

Blues harp reeds are only weakly damped, and the playing frequencies are comparatively close to the resonant frequencies of the reeds, even with bends or overbends. The reeds therefore oscillate in the steady state region "almost by themselves". During one period, kinetic energy and potential energy are continuously converted into each other, with the potential energy being equal to the elastic inner energy of the reed. Only the small energy losses per oscillation have to be replaced by the pressure force. Thus, it is understandable that comparatively small pressure forces are sufficient to maintain the oscillation of a blues harp reed.

Since the pressure forces are in fact considerably weaker than the restoring forces of the reeds (section1.3) and thereby oscillate approximately in antiphase (section 1.4), there can be no question of pressure force and restoring force acting in any sense "alternately" on the reed. The slight phase shift between pressure force and elongation is sufficient to replace the energy losses in the feedback loop and to allow a playing frequency that deviates from the resonance frequency of the respective reed.

2 Bernoulli equation, but not Bernoulli force

2.1 Bernoulli equation ...

The simplest variant of a Bernoulli equation relates to a streamline of a stationary, frictionless, incompressible fluid which is not under the influence of a gravitational field [KC 16]. If $i = 1, 2$ denotes two locations on the streamline with the static pressures p_i and the velocities v_i, then with constant density ϱ, the following equation applies:

$$p_1 + \frac{\varrho}{2} v_1^2 = p_2 + \frac{\varrho}{2} v_2^2 \tag{13}$$

The Bernoulli equation (13) compares the velocities and static pressures at locations 1 and 2, where $v_2 > v_1$ correlates with $p_2 < p_1$.

Since the pressures p_1 and p_2 act at different points, the difference $p_2 - p_1$ makes no sense at the first glance (it is the local pressure gradient that acts on the air flow along the streamline). If, however, at point 2 the air flow is limited by a wall with pressure p_1 acting on its outside, then perpendicular to this wall a "suction force" coming from the pressure difference $p_2 - p_1 < 0$ pulls the wall towards the air flow: the *Bernoulli force*.

2.2 ... but better not "Bernoulli force”

Bernoulli’s equation (13) is valid for each of the two reeds in a channel of a blues harp. On the left side there is the pressure p_M in the player’s mouth, which is a superposition of a constant positive or negative pressure generated by the lungs with acoustic fluctuations when playing the blues harp.

For a *blow note*, $p_M > 0$, and the velocity of the airflow in the oral cavity can be neglected. On the right-hand side of the Bernoulli equation are the velocity v of the airflow through the slots between the reeds and the reedplate, and the external atmospheric pressure, which is set equal to zero as a reference point. Thus we obtain for a blow note:

$$p_M + \frac{\varrho}{2} \cdot 0^2 = 0 + \frac{\varrho}{2} \cdot v^2 \tag{14}$$

The airflow through the slots moves along the rims of the reedplate and the reeds and is almost perpendicular to their surfaces. The term "Bernoulli force" is therefore misleading, at least for laypersons, since it is usually associated with an air stream grazing along a surface and exerting a suction effect on the surface. However, there is no air flow along the reed surfaces (([FH 03], cited in [MB 07]). The force originating from the pressure difference $\Delta p = p_M - 0 = p_M$ on both sides of the reeds is a "Bernoulli force" only in the formal sense that it can be described by a Bernoulli equation. If one would follow the illustrative picture of a Bernoulli force associated with an air stream sweeping along, "Bernoulli forces" would be exerted on both *lateral rims* of a reed, and the vector sum of these forces on the reed would result in zero.

For a *draw note,* the air accelerated to velocity v in the slits exits into the oral cavity, where the pressure is $p_M < 0$. At the entrance of the slits, the air velocity is approximately zero, and the pressure is nearly equal to the atmospheric air pressure. The Bernoulli equation is now:

$$p_M + \frac{\varrho}{2} \cdot v^2 = 0 + \frac{\varrho}{2} \cdot 0^2 \tag{15}$$

Again, at the location of greatest velocity v, the static pressure in the jet and the ambient pressure (this time the pressure in the mouth) are equal, and for the pressure difference Δp between the two reed surfaces, the following applies again:

$$|\Delta p| = \frac{\varrho}{2} \cdot v^2 \tag{16}$$

Thus, the external force acting perpendicular to the surfaces in the equations of motion of the reeds can be obtained from a Bernoulli equation in any case. The associated "Bernoulli forces" in a vivid sense would act on the lateral rims of the reeds and compensate each other.

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