Measurement of relaxation time and damping of a harmonica reed

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Abstract

Physical studies on playing the blues harmonica model the reeds as harmonic 1-point oscillators. The damping constant is needed for the equation of motion of a reed. In this paper two methods are presented to determine this damping constant without expensive laboratory equipment. An exemplary investigation is made of the draw reed in the fourth channel of a blues harmonica in C. In fact, after a transition phase, the decaying reed oscillation can be described as a monofrequent falling exponential curve. It can be seen that both methods, which are fundamentally different in their approach, provide the same results for frequency and relaxation time within this range, which indicates that they are indeed suitable for measuring the damping constant.

1 Relaxation time and damping

For a damped harmonic 1-point-oscillator we distinguish, following [CK 08, CK 16] beetween *resonance frequency* $\omega_0 = \sqrt{k/m}$ (with spring constant k and mass m) and *eigenfrequency* ω_E. Under the influence of a periodic external force the system oscillates with maximal amplitude at its resonance frequency ω_0 mit maximaler Amplitude. If the oscillator is "plugged" ($x_0 > 0$ und $v_0 = 0$), a damped oscillation starts with a pseudo-frequency, which is equal to its eigenfrequency ω_E.

While the oscillations of a plugged reed are decaying there is no external force. The following then applies to the deflection x:

$$\ddot{x} + 2r\dot{x} + \omega_0^2 x = 0 \tag{1}$$

The constant r describes how much the oscillation is damped. The solution of this equation is:

$$x = \frac{x_0}{\cos\varphi} \cdot e^{-r \cdot t} \cdot \cos(\omega_E t + \varphi) \tag{2}$$

with pseudo-frequency ω_E and phase shift φ:

$$\omega_E = \sqrt{\omega_0^2 - r^2} \tag{3}$$

$$\cos\varphi = \sqrt{1 - \left(\frac{r}{\omega_0}\right)^2} \tag{4}$$

After the *relaxation time* τ has elapsed, the oscillation has decayed to$1/e$ times the initial displacementx_0. Consequently, the constant r from equation (1) and the relaxation time τ are reciprocal to each other (which motivates the factor 2 in the oscillator equation):

$$r = \frac{1}{\tau} \tag{5}$$

The constant r is called the *decay constant* (*damping constant, damping coefficient*). The damping coefficient r is related to the width of resonance curves in forced oscillations, which are described by the *quality factor* Q .

Quality factor Q , relaxation time τ, resonance frequency ω_0 and damping constant r are related as follows:

$$Q = \omega_0 \cdot \tau = \frac{\omega_0}{2r} \tag{6}$$

A quantity also used as a measure of damping is χ, which is also referred to as the *damping constant*:

$$\chi = \frac{2r}{\omega_0} = \frac{1}{Q} \tag{7}$$

Thereby "damping constant" can alternatively mean r, $2r$ or χ by different authors. Furthermore, $D = r/\omega_0$ is also called "Lehr's damping measure" (Lehrsches Dämpfungsmaß), thus $\chi = 2D$.

2 Measurement of damping

2.1 Indirect measurement methods

Without external forces, the initial conditions $x_0 > 0$ and v_0=0 are satisfied for the solution (2) of equation (1) at each turning point (with appropriate direction of the x-axis). Thus, from a measurement of $x(t)$, one can determine the damping constant using (1).

Oscillations of a blues harp reed without an external force can be generated by plucking the reed or by playing a note and taking the instrument away from the mouth as quickly and as far as possible. Without expensive laboratory equipment, one can obtain information about the oscillation process by recording the emitted sound in the first case (plucking) or by evaluating signals from a pickup module (see section 2.1.2) in the second case (taking the harp away from the mouth). We applied both approaches to a reed in channel #4 of a C harp.

2.1.1 Sound recordings

The radiated sound after plucking the reed was recorded in the near field with a recorder *ZOOM-H4n* and evaluated with *Audacity* (open source). An evaluation of the resulting wave curve assumes that the voltage fluctuations at the recording microphone are proportional to the reed displacement $x(t)$. This seems plausible if one imagines the moving reed surface as a kind of loudspeaker membrane[1].

2.1.2 Turboharp ELX

ELX [ELX] is a pickup module that converts the movements of the reeds into AC voltage using reflective optical sensor s[ROS]. The output of the *ELX* is connected to the line-in input of the *ZOOM-4n* recorder. The characteristic curves of optical sensors are not strictly linear, and additional electronics are built into the *ELX*, so it is ultimately a black box with unknown behavior. However, a frequency analysis of recorded wavedata shows the behavior expected from bluesharp reeds: With about $30dB$ distance to the next harmonic, the signal is sinusoidal to a good approximation, just as the reed oscillation is according to [Mi 99]. As long as one is only interested in amplitudes and oscillation durations (but not, for example, in phases), one can assume that a wave file recorded in this way reproduces the reed movement approximately proportionally.

2.1.3 Evaluation

Assuming that the voltage fluctuations displayed on the screen are proportional to the reed deflection, the approximate determination of relaxation times τ or half-lives $T_{1/2}$ becomes possible. Conveniently readable on the screen for both methods is the half-life $T_{1/2}$, from which the damping constant r can be calculated:

$$r = \frac{ln2}{T_{1/2}} \tag{8}$$

Because of the low damping, eigenfrequency and resonant frequency are practically the same, so that the damping constant χ can also be obtained from the determination of the pseudo eigenfrequency f_E using (7) :

$$\chi = \frac{2r}{2\pi f_E} = \frac{ln2}{\pi f_E T_{1/2}} \tag{9}$$

[1] „A loudspeaker is an electroacoustic transducer ... the waveform in the acoustical system being substantially equivalent to that in the electrical system.“ [Ol 67, S. 336] „A microphone is an electroacoustic transducer ... the waveform in the electrical system being substantially equivalent to that in the acoustical system.“ [Ol 67, S. 325]

2.2 Results

2.2.1 Plucking

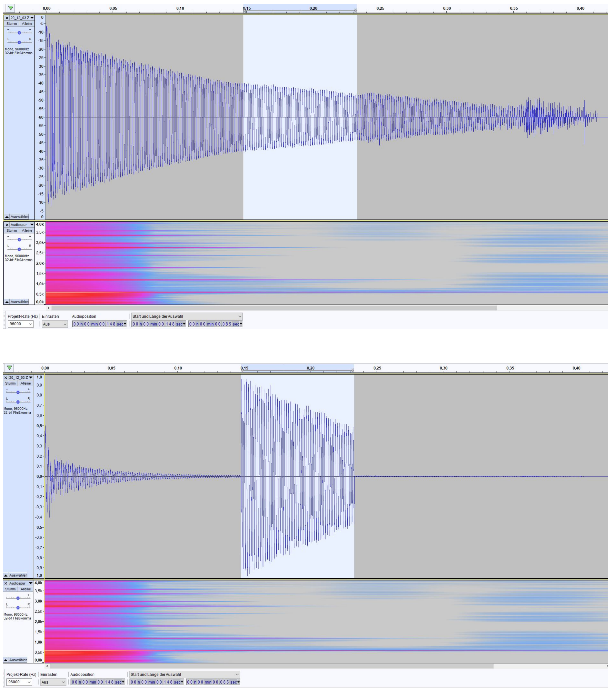

Figure 1: Plucking a reed (reed D in channel #4 of a C-Harp). Top: Wave in logarithmic representation and spectrum. Bottom: The same wave in linear representation and spectrum, the marked section was represented amplified to determine the half-life.

Fig. 1 shows the representation of the wave file recorded when plucking a reed D in channel #4 of a C-Harp using *Audacity*. The spectrogram shows that after plucking, the reed does not initially behave like a monofrequency oscillator. By plucking with a paper handkerchief stretched between the fingers, this range could be kept comparatively small. The logarithmic plot (above) shows two successive intervals of exponential decay. Again looking at the spectrogram, it is clear that the exponential curve (2) assumed in chapter 1 should be a good approximation for the tongue oscillation in the second section, where the spectral line at $598Hz$ dominates with $30dB$ separation over the only other spectral line. To measure the half-life $T_{1/2}$, the region marked in Fig. 1 was boosted in linear representation in this section (bottom). With $T_{1/2} = 85ms$ and $f_E = 598Hz$, (9) gives the damping constant $\chi = 0,0043$.

2.2.2 Taking the harp away from the mouth

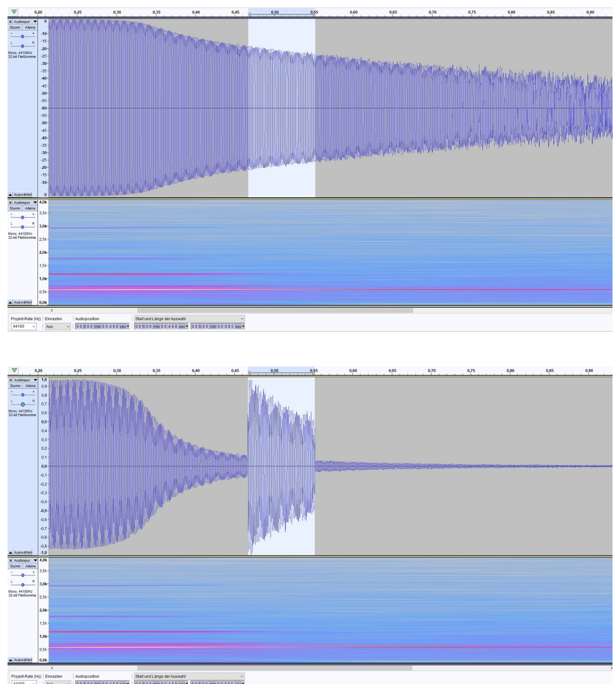

Figure 2: Play normal draw note on channel #4 of a C-Harp and quickly remove instrument from mouth. Top: Wave in logarithmic representation and spectrum. Bottom: The same wave in linear representation and spectrum, the highlighted section was represented amplified to determine the half-life.

Fig. 2 shows corresponding screenshots, where first a normal draw note was played on channel #4 of the same C harp as in section 2.2.1, and then the instrument was abruptly taken away from the mouth. An exponential decay apparently begins in the marked region, where the spectrogram shows a dominant spectral line at $597Hz$. With a read-off half-life of

$T_{1/2} = 85ms$ and with $f_E = 597Hz$, (9) again gives the damping constant $\chi = 0.0043$.

2.2.3 Comparing the results

In both experiments, the reed indeed seems to oscillate undisturbed and monofrequently after some time, thus behaving like a harmonic oscillator described by the equations (2) and (5).

With the two fundamentally different methods, comparable results for half-life and frequency were obtained even when the experiments were repeated and the half-life was measured at other points in the exponential decay range. This indicates that these procedures are basically suitable for the measurement of the damping constant.

References

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