Blues harmonica = “Mississippi saxophone”?

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Abstract

This article collects experimental findings on playing the blues harp, which a physical model should thus be able to explain. The leitmotif will be the question whether and how suitably defined resonance properties of the vocal tract influence playing frequencies.

When playing blow or draw notes on a channel of a blues harp, the airflow through the player’s respiratory tract and through the instrument together with the closing and the opening reed in the channel perform self-excited oscillations with a common playing frequency. This is not obvious, instead two closing reeds in the channel generate self-excited oscillations close to their respective eigenfrequencies.

Advanced players are able to change the pitch of normal notes on the blues harmonica by appropriate variations of the vocal tract geometry. MRI images [ESBRH 13] show a narrow constriction between tongue and palate together with a well-defined front cavity volume. A measured [BAB98] correlation between playing frequency of various bend notes and front cavity volume suggest that constriction and front cavity together form a Helmholtz resonator, whose resonance frequency has a decisive influence on the periodicity of the self-excited oscillations of airstream and reeds. On the contrary, measurements in [J 87] with a blues harmonica excited by a tube resonator suggest a less intuitive interaction between resonator and reeds. For the player’s vocal tract as a resonator, the phase angle of its admittance is postulated to define playing frequency.

Normal notes sound with any arbitrary relaxed embouchure, so they should be explainable without reference to a resonator. On the other hand, it is possible to bend a normal draw note continuously down, so physical models for normal and for bend notes should also be connected by continuously changing some common parameter.

It is possible to bend draw notes on two or three neighbouring channels or even octaves (using tongue split) simultaneously with one common embouchure. Changing rapidly between inhaling and exhaling, one can play a draw bend and an overblow on the same channel with one embouchure, whereas playing frequencies lie one third apart. On the lower channels, an overblow and a blowbend can be played simultaneously, although the blowbend sounds significantly louder and always prevails over the blowbend during the transition to a single note.

The latter listed experiments contradict the intuitive notion that the playing frequencies of bends and overbends are simply determined by a single resonant frequency of the vocal tract near the playing frequency. On the other hand, comparable geometries of the oral tract occur when speaking vowels, whistling, singing overtones, or bending notes on the saxophone, where they have been shown to act via resonant properties of the enclosed air volume. It would therefore be desirable to conduct experiments that can simultaneously record the geometry of the vocal tract and the resonance behavior of the air flowing through it, the oscillations of the two reeds and the sound emitted when playing the blues harp.

1 Normal notes, bends and overbends

A blues harmonica (blues harp)[1] is a diatonic harmonica with ten channels [BAB98]. In each channel there is a blow reed and a draw reed. The reeds are riveted to the reedplates and can oscillate freely through openings in the reedplates (free reeds). The blow reeds are located on the underside of the upper reedplate, i.e. inside the channels. The draw reeds are located on the underside of the lower reedplate, i.e. outside the channels. For the instrument to be playable, the reed tips must protrude slightly from the reedplate (clearance gap). Blowing into the channel, the blow reed initially behaves (as long as it does not enter the opening in the reedplate) as a closing reed and the draw reed behaves as an opening reed. When one starts drawing the blow reed behaves as an opening and the draw reed as a closing reed.

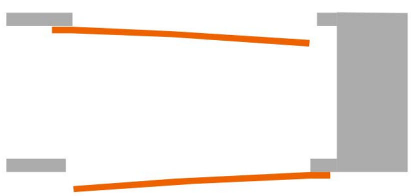

Figure 1: The two reeds inside a channel

Playing the harmonica with relaxed embouchure results in *normal* blow or draw notes, with mainly the *closing* reed in the channel oscillating. The corresponding frequencies lie somewhat below the eigenfrequencies of the reeds. Eigenfrequencies (natural frequencies) can be heard and measured by plucking the reeds. Due to the small damping of a harmonica reed, its eigenfrequency is nearly identical with its resonance frequency.

In channels #1 to #6 the reed with the higher pitch is a draw reed, in channels #7 to #10 this reed is a blow reed. By appropriately changing the geometry of the respiratory track, one has the subjetive feeling of lowering (bending) the playing frequency of the normal note with the *higher* pitch in the channel. In fact *both* reeds are involved in a bend. The deeper the bend, the more the opening reed oscillates. The playing frequency of *bend note*s lies between the eigenfrequencies of the two reeds in the channel.

Trying to bend the normal note with the *lower* pitch in the channel, the tone might instead "pop up" a scant minor third (or there will be no tone at all). In contrast to this subjective perception the tone is generated by the *opening* reed which oscillates somewhat above its eigenfrequency. In analogy to bend notes such notes are called *overbends*. Advanced players can hit such tones in a controlled manner and can bend them up to still *higher* frequencies.

To sum it up, one can play *draw bends* and *overblows* on channels #1 to #6 as well as *blow bends* and *overdraws* on channels #7 to #10.

2 The blues harmonica - a "Mississippi saxophone"?

The most accurate macroscopic physical description of the air flow from the players lungs through the instrument to the surroundings (for blow notes) or vice versa (for draw notes) would be provided by the Navier-Stokes equations. Unfortunately, even our best computers could not solve those equations - the boundary conditions given by the player's respiratory tract and the instrument are just too complicated.

Without computer-generated visualizations, we are left with conventional approaches to intuitive physical understanding. A paradigm for the "Mississippi saxophone" - as the harmonica was sometimes called in former times - could be the saxophone. The oscillating reed in the mouthpiece of a saxophone together with the oscillating air in the bore define a feedback loop. The frequency of the resulting self-excited oscillations is dictated by the resonance frequency of standing waves in the bore. Those standing waves are not perfectly enclosed at the open end, allowing sound to "escape". The overpressure generated in the player's lungs helps to compensate for energy losses.

[1]In the following we assume a standard blues harp in Richter tuning

These intuitive ideas have been refined and deepened over the years [F 79, CK 08, CK 16].

Of course there are essential differences between the blues harmonica and the saxophone: there is no bore, there are two free reeds in a channel instead of one beating and heavily damped reed in the mouthpiece, sound is generated by the jets through the slits between reeds and reedplate. But two key ideas could perhaps carry over: *feedback* and *resonance* (with the player's oral cavity as resonator, see chapter 4.1).

3 Feedback

3.1 Self-excited oscillations of a single free reed

If you are lucky, you can model a spatially distributed, complicated physical system, such as the airflow described by the Navier-Stokes equations, by discretized, bounded building blocks characterized and connected by simple (differential) equations (lumped-element model). This together with further simplifications and approximations is the general idea behind the above mentioned intuitive models of a *saxophone*: An abstract one-point harmonic oscillator (the reed) and an abstract linear single mode oscillator (the air in the tube) interact at an abstract border (somewhere at the end of the mouthpiece) through the values of scalar-like volume flow and pressure oscillations (one needs two quantities to encapsulate the idea of an "air flow"). These values of volume flow and pressure at the border represent the state of the oscillating air in the tube. At a first glance it seems intuitively clear that the air flow (i. e. the air in the mouthpiece) influences the reed (blowing at the reed causes it to move) and that the reed influences the air flow (the reed can even close the opening, so there is no flow at all). But things are not quite as simple, because air flow is described by two quantities, usually by volume flow and pressure. Together with reed opening these are *three* variables, which are linked by *three* equations [CK 08, CK 16].

Given two such correlated systems together with an external energy source (the player), random disturbances can result in periodic behaviour. At first values of volume flow, pressure and reed opening will increase exponentially, then this growth will be limited by additional conditions, and finally harmonic oscillations may establish.

For a single free reed instrument (an accordion, a chromatic harmonica, or a blues harmonica with one of the reeds in the channel covered and fixed by some tape) let us consider the reed, represented by the elongation x of an abstract one-point oscillator, and the air flow through the reed gaps. Asuming equal jet velocity u all over the gaps and homogenous pressure difference Δp between both sides of the reed one may think of an air flow described by u and Δp at one point in an abstract 1-D space. Again it seems intuitively clear that the two systems (reed and air flow) are correlated. The harmonic oscillator will be submitted to frictional forces $\sim \dot{x}$ and an external force $\sim \Delta p$. In the steady state region of a self-excited oscillation these two forces must cancel. This can only be the case with a suitable delay between elongation x and pressure fluctuation Δp in frequency space.

Explaining such a delay is part of an intuitive understanding of the feedback process. In the case of the saxophone this delay is caused by the resonator. But feedback in an air-driven instrument is possible without any resonator. In the *accordion*, the inertia of the inflowing air in the upstream region creates the necessary delay between reed elongation and pressure fluctuation at the reed [RCM 05]. As you might expect in the absence of a resonator, the playing frequency is practically identical to the eigenfrequency of the reed.

Normal notes on the blues harmonica sound with nearly every relaxed embouchure - otherwise the instrument would be hard to sell. An oral cavity without significant constrictions and without pronounced resonances ensures a comparably undisturbed airflow. Asuming that normal notes are mainly generated by a feedback process between the respective closing reed and the airstream, the resulting self-excited oscillations could be understood in analogy to the oscillation of single accordion reeds.

Summing up, bend and overbend notes notes might be modelled in analogy to the saxophone, whereas normal notes might be modelled in analogy to the accordion. Mind, however, that one can bend normal draw notes on the lower channels continuously down to lower frequencies by continuously changing one's embouchure. A model explaining normal draw notes should therefore turn out as the limit case of a model for draw bends with the vocal tract geometry appearing in *both* models.

3.2 Bending and synchronization of two reeds

Fig. 2 shows the transient behaviour of the oscillations of the two reeds in channel #3 of a C-Harp playing a half-tone bend[2] [Fö 19]. The combined system of the air flow through oral cavity and instrument into the player's lungs together with the two reeds of different eigenfrequencies needs less than five hundredths of a second for perfect self-organization: The rectangular frame marks 39 (quasi-) periods in a steady state with both reeds oscillating with the same frequency of $456Hz$ with a relative deviation of only $0,024\%$.

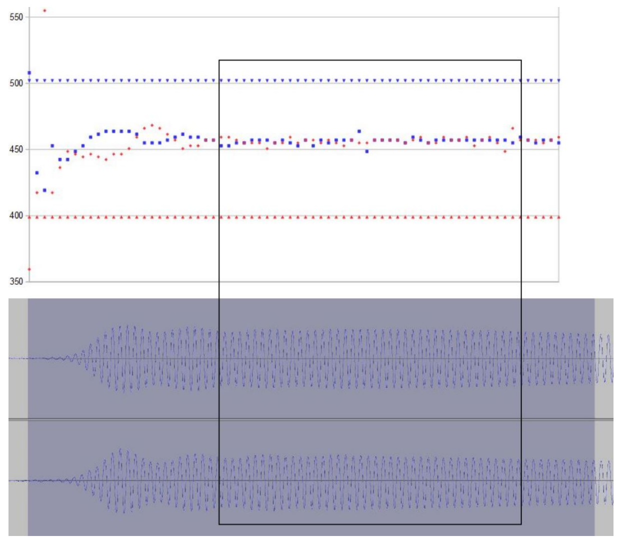

Figure 2: Transient behaviour of the two reeds in a channel, playing a half-tone bend Bb on channel #3 of a C-harp. Above: Frequencies determined by measuring (quasi-) period lengths in time domain for the indicated region. Red: blow reed, blue: draw reed. The normal draw note B has a frequency of $399Hz$, the normal bow note G a frequency of $502Hz$. Below: Corresponding oscillations of both reeds in time domain in linear representation (above: draw reed, below: blow reed).

4 Resonances

4.1 Vocal tract resonances

In contrast to the saxophone, the blues harmonica has no resonator volume of its own[3]. However, a reasonable candidate for a resonator should be the player's respiratory tract together with the reed channel. Referring once more to the saxophone, the influence of the embouchure on playing frequency can be modelled within the usual resonator model by treating vocal tract and bore impedances in series [CSW 11]. Subjective experience of the author shows that one can bend notes on a tenor saxophone with configurations of the vocal tract used for bending notes on the harmonica.

MRI images [ESBRH 13] confirmed the subjective perception of players that the tongue position has a significant influence on the formation of bends and overbends. The most important point seems to be a constriction between the tongue and the palate, which for high tones (up[4] to about Dis 6 $= 1245Hz$) is located far forward in the mouth as with the vowel [y], and for low tones (down to about G3 $= 196Hz$) is located far back in the throat as with the vowel

[2] Periodicity and frequency of harmonica reed motions were measured with help of a *Turboharp ELX* [ELX] (see chapter 5.1). Frequencies were determined by measuring (quasi) period lengths of corresponding wave files in time domain as explained in [Fö 19].

[3] For very high notes, the volume of the reed chamber could come into play.

[4] In former times you could buy blues harmonica from G (lowest tuning) in half steps up to Fis (highest tuning). D 6 was thus the highest attainable draw bend on channel #6 of a Fis-harmonica. There were also available extra tunings in "low C". G3 on channel #2 of such a harp is the lowest note the author is able to bend.

[u]. Tones in between are playable accordingly with tongue positions in between.

Vowels in speech production are characterized by *formants*, and formants are caused by vocal tract resonances. Following Fant [F70], the constriction between tongue and palate defines a back cavity and a front cavity volume. Together with the lips channel, this constriction and the various volumes give rise to different kinds of standing waves and Helmholtz resonances, which can explain the observed formant frequencies. The evaluation of MRI images confirmed this intuititive approach [SMWS 92] which is refined by calculation of formants with the help of transmission line techniques [K 00, MSBA 07].

The tongue positions described above also occur when whistling. *Whistling* can be explained as self-excited oscillation with the combination of front cavity and lip channel as a Helmholtz resonator [F70, APM 18]. The amplification of the high pitch of the biphonic sound in *overtone singing* is caused by the coincidence of resonances and formants [KNRS 01, KNR 03, ENS 20].

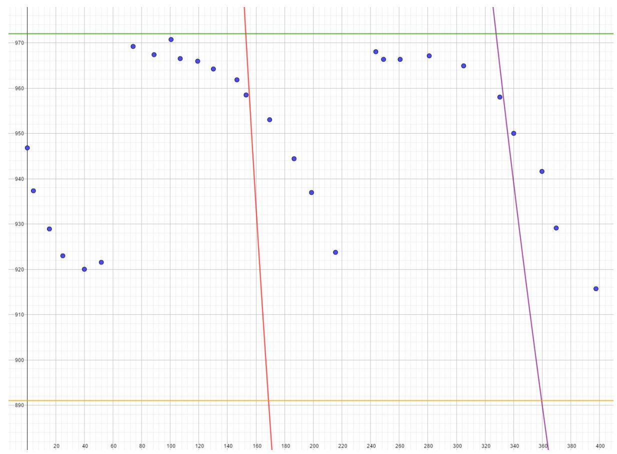

Figure 3: [J 87] Blow note frequencies (Hz) on channel #8 of a G-harp with blow reed B5 and draw reed A5 for a given blowing pressure as a function of tube length (mm). Horizontal lines: free reed frequencies. Nearly vertical curves: Resonance frequencies of a tube of length L with wave lengths $L/2$ (red) and L (violet).

4.2 Tube resonances and bending

Johnston [J 87] connected channel #8 of a G-harp to a cylindrical tube and measured the playing frequency of a blow note as a function of blowing pressure and tube length. Fig. 3 shows measured frequencies for a given pressure as a function of tube length. The nearly vertical lines depict (inspired by [Co 13]) resonance frequencies of a cylindrical tube of length L connected with a reed channel of length $2cm$. For a tube nearly closed at *both* ends (as realized in Johnston's experimental setup) there are standing waves of wavelength $(L + 2cm)/2$ and $L + 2cm$. Obviously, playing frequencies depend significantly on tube length but are *not* simply equal to the resonant frequency of the tube. For very small tube lengths there is no corresponding standing wave at all.

Which kind of standing waves could explain draw bends for a human player? A draw bend in the mid frequency range is G3 on channel #2 of an A-harp with frequency $f = 392Hz$. The area of the glottis opening during normal breathing equals about $2cm^2$ [Sch 12], while the reed channel is nearly closed by the reeds in the channel. Assuming a $\lambda/4$ standing wave between glottis and the end of a reed channel for a draw bend G3 would require a tube length of $22cm$. Bending on channel #1 of the A-harp would amount to a tube length of $36cm$, which seems unrealistic.

The role of the palatal constriction in the standing wave picture would only be to increase tube length by forcing the standing wave in a kind of "detour". As we will see in chapter 4.3 a more natural model which also provides us with more plausible resonance frequencies would be the Helmholtz resonator.

4.3 Helmholtz resonances and bending

Bahnson and Antaki [BAB98] conducted an experiment with a professional player (Howard Levy) who played different bend notes and then held the configuration of the vocal tract while the oral cavity was filled with water and the required volume was measured. For draw bends on channels #1 and #4 on harps in different keys they found the values depicted as blue dots in fig. 4. As mentioned by the authors, these values are the volumes of the anterior oral cavity rather than the total oral volumes[5].

Millot[6] mentioned [M 99, p. 94, footnote 12] the idea to deal the constriction together with the front cavity as a Helmholtz resonator.

Figure 4: [BAB98] Draw bending on channels #1 and #4 on various blues harmonicas: Volume of oral front cavity ($1 = 100cm^3$) as a function of pitch (n denotes the number of half-tone steps above Ab3).

Let $f_0 = 208Hz$ denote the frequency of the lowest pitch Ab3 in Fig.4, f_n the frequencies of the higher notes with n ($n = 0, 1, ...$) denoting the number half-tone steps relative to Ab3, V_n the front vacity volume corresponding to f_n, $c = 34000cm$ the speed of sound, $S = 0.8cm^2$ the area of the constriction and $L = 4cm$ its length (values for the constriction taken from [ESBRH 13]). Then we have:

$$f_n = 2^{-\frac{n}{12}} \cdot f_0 \tag{1}$$

$$f_n = \frac{c}{2\pi}\sqrt{\frac{S}{L}} \cdot V_n^{-\frac{1}{2}} \tag{2}$$

Equating (1) and (2) results in:

$$V_n = \frac{c^2 S}{4\pi^2 L f_0^2} \cdot 2^{-\frac{n}{6}} \tag{3}$$

With the above mentioned values we get the green curve depicted in Fig. 4. Inserting $L = 3.5cm$ defines the orange curve.

Fig.4 might be interpreted as a strong indication for a correlation between Helmholtz resoncance and bending on the blues harmonica. But as a warning, it should be noted that the shape of the curves in Fig.4 depends sensitively on the values L and S for length and area of the constriction, and of course we don't know whether these values were comparable for the two players (Barrett and Levy) in two different experiments, especially because Barrett played a whole tone bend, while Levy played only half tone bends.

It would be interesting to know the volume of the front cavity for Barrett's whole tone bend on channel #3 of a C-harp in [ESBRH 13] and to insert it in (2). The authors mention volume data in [GEBR 13], but (to my knowledge) didn't publish or evaluate them.

4.4 Johnston's theoretical model

4.4.1 Phase shift determines frequency

The theoretical part of [J 87] is based on [F 79]. Nowadays these models live in the field of linear stability analysis (see [FöF, FöJ]) .

In analogy to electric one-port oscillators, a negative-resistance "generator" (the two reeds in a channel switched[7] in parallel) intereracts with a resonator.

[5]"It is probable that actual playing volumes were larger than measured, since when water was instilled, constriction of the glossopharynx and larynx occurred in order to suppress the swallow reflex and prevent aspiration. The volume of the anterior oral cavity was found to be inversely related to pitch as modified by bending." [BAB98]

[6]Laurent Millot models tone generation on the blues harp detached from established approaches to the physics of wind instruments and without explicit reference to resonances. He traces the airflow through the vocal tract and the instrument by a series of approximating equations and solves the resulting system of equations numerically.

[7]For each of the two reeds a harmonic oscillatar equation is combined with the Bernoulli-like behaviour of the air stream through the reed slits, and after linearization the resulting admittances are added ("resistors" in parallel circuit).

Playing frequencies should only be possible for a negative real part of the admittance of the generator[8]. Bend notes are thus restricted to the frequency range between the two eigenfrequencies of the reeds. Which frequency actually sounds should depend on the phase angle Φ_{gen} of the input admittance Y_{gen} of the generator which has to match the phase angle Φ_{res} of the resonator input admittance Y_{res} in the sense that[9]:

$$-\Phi_{gen} = \Phi_{res} \tag{4}$$

Equation (4) follows from the continuity of pressure and volume flow at the boundary between generator and resonator. Since both sides refer to input admittances "looking" in opposite directions, there must be a minus sign on one side.

The red curve in Fig. 5 shows the inverse phase angles $-\Phi_{gen}(\omega)$ for the admittances of a blues harmonica (draw bends on channel #4 of a C-harp, red curve) calculated with Johnston's formulas, using eigenfrequencies (black dots on the frequency axis) $\omega_o = 3300s^{-1}$ of the opening reed and $\omega_c = 3700s^{-1}$ of the closing reed [FöJ]. These formulas are based on the notation of the complex admittances by means of a real "amplitude" which can also assume negative values. As a result, the phase angles can be chosen between $-\pi/2$ and $+\pi/2$.

The real part of the reeds admittance (not shown here) is (only) negative for playing frequencies between those two reed frequencies, which characterizes bend notes. In this region the real part of $-Y_{gen}$ is positive, so that the corresponding phase angles $-\Phi_{gen}(\omega)$ have values between $-\pi/2$ and $+\pi/2$, which remains true using the usual complex notation.

4.4.2 Two mode resonator

Tubes as well as vocal tracts have more than only one resonance frequency. Such a resonator may approximately be described by a mode expansion of its impedance $Z(\omega)$ in frequency space [CK 08, CK 16]:

$$Z(\omega) = j\omega \sum_n \frac{F_n}{\omega_n^2 - \omega^2 + j\omega\omega_n/Q_n} \tag{5}$$

with resonance frequencies ω_n, quality factors Q_n and modal factors F_n.

The phase angle Φ_{res} of the resonator input admittance equals minus the phase angle of the impedance, thus $\Phi_{res} = -\arctan \mathrm{Re}Z/\mathrm{Im}Z$. Solving (4) graphically we have to intersect the red curve for the generator in Fig. 5 with the graph of $\arctan \mathrm{Re}Z/\mathrm{Im}Z$.

The violet curve describes exemplarily the admittance of a single mode resonater and intersects the red curve at the resonator frequency $\omega = 3600s^{-1}$. This contradicts the results of Johnstons's experiments depicted in Fig. 3 with playing frequencies significantly different from tube resonance frequencies.

The blue curve in Fig. 5 is based on such an expansion with two modes $\omega_1 = 3600s^{-1}$ and $\omega_2 = 8000s^{-1}$

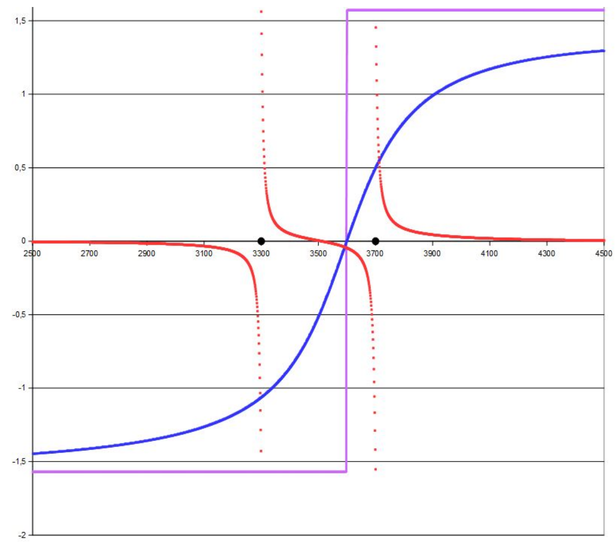

Figure 5: Inverse phase angles (definition of phase angles following [F 79, J 87]) for the admittances of a blues harmonica (draw bends on channel #4 of a C-harp, red curve), one single mode oscillator (violet) and one resonator with two modes (blue) as a function of circular frequencies ω.

[8]This condition ensures that the generator can supply the oscillating system with energy coming from DC energy supported by the player's lungs [CK 08, 3.4.3].

[9]Actually not only the phase angles, but also the amplitudes of the complex admittances should match. It is asumed that this can be accomplished by variation of the constant blowing pressure at the input of the generator.

with equal modal factors $F_1 = F_2$ and with equal quality factors $Q_1 = Q_2 =: Q$, and it will show how Johnston's phase condition could in principle work. Inside the allowed intervall the red curve is intersected at a playing frequency $\omega = 3593s^{-1}$ different from the resonator frequency $\omega_1 = 3600s^{-1}$, but only $1Hz$ less. The intersections of the blue and the red curves below and above the reed frequencies are forbidden because the real part of the admittance would be positive. Varying the higher mode frequency has practically no influence on the playing frequency, whereas the value of the quality factor is crucial for the distance between playing frequency and frequency of the lower mode. We have chosen $Q = 10$, for $Q = 100$ the resulting playing frequency $\omega = 3599s^{-1}$ would practically equal resonance frequency.

4.4.3 Tube resonator

In the Appendix of [J 87], Johnston cites Fletcher's formula [F 79, Appendix] for the phase angle Φ of the inlet admittance of an open tube with energy losses:

$$\Phi = -\arctan\left(\frac{H^2-1}{2H}\sin\frac{4\pi}{c}f\cdot L\right) \qquad (6)$$

H is a constant with values $10 \le H \le 100$ and stands for the amount of losses, f denotes frequency, L is the length of the resonator tube.

Johnston notes that the phase angle changes continuously from $\pi/2$ to $-\pi/2$ near the resoncance and predicts a similar behaviour of the phase of the admittance of the vocal tract seen from the lips. Therefore each allowed frequency should actually be playable by adjusting the required phase angle.

We will demonstrate this idea using (6), although a realistic model of the vocal tract would essentially take into account the role of the palatal constriction for bends and overbends. We will chose $H = 50$, so the constant $\big(H^2-1)/2H\big)$ in (6) can be approximated by $H/2$. Our results will not depend on the exact value of H. Due to the periodicity of the sine function there are infinitely many possibilities to solve equation (6) for tube length l. A reasonable choice will be:

$$l = \frac{c}{4\pi f}\left(\arcsin\left(-\frac{2}{H}\tan\Phi\right) + 2\pi\right) \qquad (7)$$

As in the previous chapter 4.4.2 we will have a look at channel #4 of a C-harp. Figures 6 and 7 show the tube length providing a phase angle as depicted in Fig. 5 (red curve) as well as the corresponding difference between playing frequency and resonance frequency of such a tube as a function of angular frequency ω for the interval between the eigenfrequencies $\omega_o = 3300s^{-1}$ and $\omega_c = 3700s^{-1}$ of the opening blow and the closing draw reed. The real part of the admittance (not depicted here) is negative exactly for the intervall between these eigenfrequencies, which is therefore the region of admissable playing frequencies.

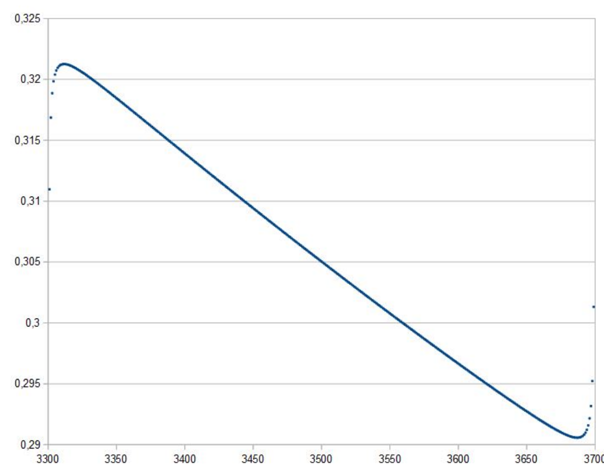

Figure 6: Length (in m) of a tube whose phase angle of admittance matches the phase angle in Fig. 5 (red curve on the intervall from $\omega_o = 3300s^{-1}$ to $\omega_c = 3700s^{-1}$, the eigenfrequencies of the blow resp. the draw reed in the channel) as a function of angular frequency ω.

Fig. 6 allows bend notes with playing frequencies which can be continously lowered, starting somewhere below the eigenfrequency of the draw note in the channel, by gradually raising tube lengh from about $29cm$ to $32cm$. As can be seen in Fig. 7, the

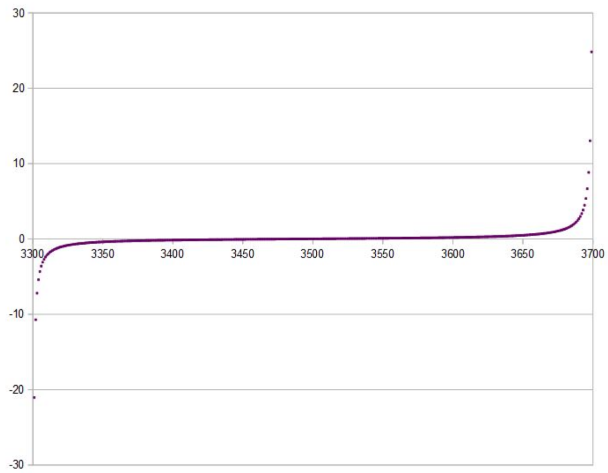

Figure 7: Difference between playing frequency and resonance frequency (in Hz) for tube lengths depicted in Fig 6 as a function of angular frequency ω.

difference between playing frequency and resonator frequency could hardly be appreciated by a listener.

Summing up, this oversimplified model says little about the importance of Johnston's phase angle condition for playing frequencies beyond the more intuitive statement that resonance frequency of the resonator dictates playing frequency. In view of Johnston's experiment (see chapter 4.2) this result is somewhat surprising.

4.4.4 Realistic phase shifts

Let $\Phi_Z(\omega)$ resp. $\Phi_Y(\omega)$ denote the phase angle of the impedance Z resp. admittance Y looking into the player's mouth. Measured (and therefore realistic) curves of $\Phi_Z(\omega)$ for expert saxophon players adjusting 'ah'-like or 'ee'-like vowels are shown in [L 15, Fig. 2]. It would of course be interesting to have corresponding measurements of $\Phi_Y(\omega)$ looking into a harmonica player's mouth, or at least numerical calculations, t.e. on the basis of MRT images as in [ESBRH 13]. To my knowledge no such data exist, so there is no realistic test of Johnstons theory.

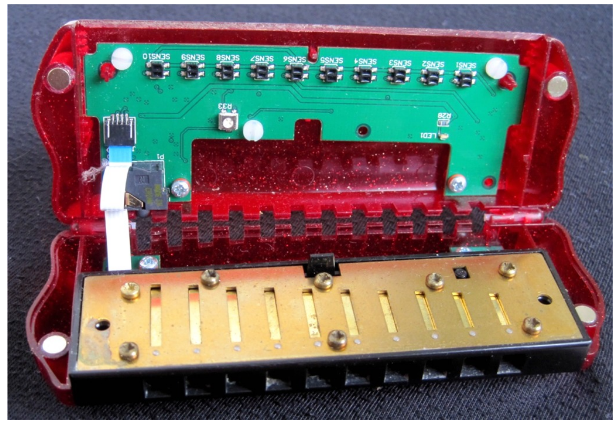

Figure 8: *ELX* opened, optical sensors at the top of the image.

4.4.5 Asian free reed instruments

In [DC 06] a phase condition is applied to Asian free reed mouth organs (instruments with one single symmetric reed coupled to a resonator). There are indeed examples where the playing frequency deviates significantly from the resonance frequency but can be predicted by the phase condition.

5 Experimental observations

5.1 Experimental setup

In chapter 5 we will present recorded sound spectra of the emitted sound of a blues harmonica as well as recordings of the output of an *ELX* (a kind of "pickup" for blues harps, see below) which mirror the spectrum of reed motion. The recordings were made with a *ZoomH4n* (96kHz, 16bit), using its built-in mics for sound recordings, and evaluated (Hann-Window, 2048bit window length) using *Audacity* (open source, freeware). All examples are played by the author.

The emitted sound of a blues harmonica is linked to jet velocity through the gaps between reeds and reed plate by Lighthill's analogy [RCM 05]. Jet velocity and pressure fluctuations in the reed channel are related by a Bernoulli equation, and pressure fluctuations in the reed channel are connected with pressure fluctuations in the player's mouth. Thus, recorded sound spectra will provide some insight into the resonance properties of the vocal tract.

The periodicity of reed motion can be evaluated with help of a *TurboHarp ELX* [ELX] (see Fig. 8) which transforms the displacement of the reed surfaces into voltage fluctuations using optical sensors and additional built-in electronics. The output generated by the two reeds in a channel can be evaluated separately as left and right channel of a stereo signal. In spite of unknown nonlinearities (the *ELX* as a "black box"), information about periodicity of reed motion and thus about playing frequencies is assumed to be reliable.

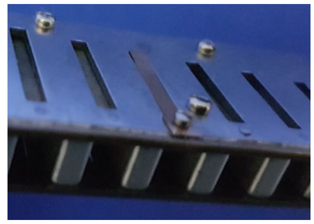

Figure 9: Harp with one blow reed on the outside.

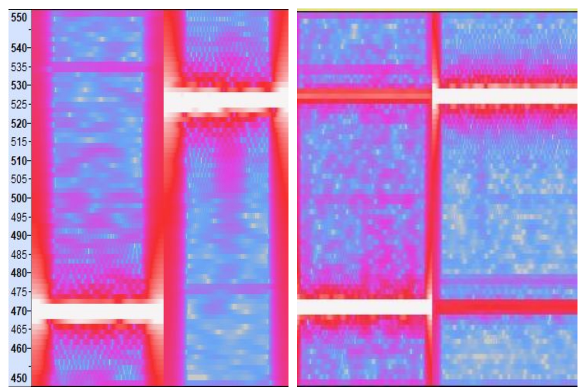

Figure 10: Two closing reeds, oscillations signals recorded with *ELX*. Left: Frequenices of single reed movements (partner reed is taped). Right: Frequencies for both reeds oscillating simultaneously.

5.2 Closing and opening reed

As mentioned at the beginning, each reed chamber of a blues harp is "inhabited" by a closing and an opening reed of different eigenfrequency. Measurements of the reed movements reveal perfectly synchronized common oscillation frequencies of both reeds for all kinds of notes (normel, bend, overbend). This observation is important for the discussion of feedback mechanisms involving those *two* one-point oscillators.

What, if there are two closing (or two opening) reeds in the same channel? Fig. 9 shows a Hohner Bb-harmonica (without cover plates) with the blow reed Bb5 on channel #4 mounted on the outside, so that both reeds in channel #4 act as closing reeds when the player is drawing. As single reeds (the second reed taped), the two reeds Bb4 and C5 oscillate with frequencies $469Hz$ resp. with $526Hz$ (left part of Fig. 10) In combination, the maxima in the spectra were $471Hz$ resp. $528Hz$ (right part of Fig. 10). Both spectra showed peaks (about $25db$ weaker) at the frequency $528Hz$ resp. $471Hz$ of the partner reed. Obviously interaction with the airflow results for both reeds in strong self-excited oscillations near their eigenfrequencies and in forced weak oscillations with the eigenfrequency of the partner reed.

A simple resonator model in frequency space asserts *a priori* a single common oscillation frequency for all parts of the combined system. This is actually true for all kinds of notes on a blues harmonica. A complete model should *explain* how this emerges from the fact that one reed is closing whereas the second reed in the channel is an opening reed.

5.3 Bending on adjacent channels

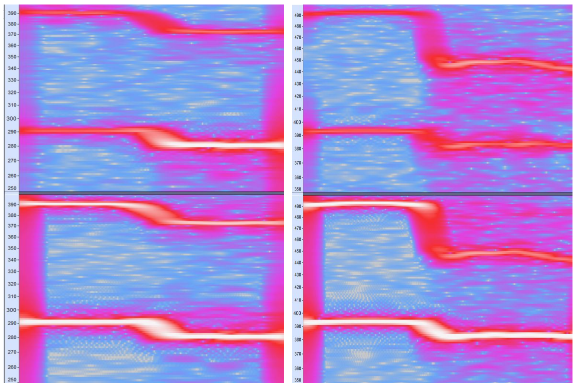

Figure 11: Simultaneous bends on channels #1 and #2 (left) and channels #2 and #3 (right), summation signal of oscillations recorded with help of *ELX*. Excerpts from spectrograms for the oscillations of the blow reeds (upper half of each picture) and the draw reeds (lower half). Left: Bending down D4 on channel #1 by 61$cent$ and G4 on channel #2 by 58$cent$. Right: Bending G4 by 36$cent$ and B4 (on channel #3) by 63$cent$.

It is possible to bend neighbouring draw notes on the lower channels of the blues harmonica simultanously. Playing a tongue split on channels #1 to #4 (the channels #2 and #3 are covered by the tongue) I am able to bend an octave. Fig. 11 and 12 show excerpts from spectrograms of the reed movements. The upper half of each picture shows the maxima in the frequency plot of the summation signal of the blow reeds in the two or three channels, the lower half comes from the draw reeds. Starting near the eigenfrequencies of the reeds the playing frequencies are bended down (bending intervalls in *cent*, as measured in frequency plots, are indicated in addition to Fig. 11 and 12).

Asuming a 1-1 correspondence between resonances of the respiratory tract and frequencies in the interval between 294Hz (D4) and 587Hz (D5), there should be one such resonance responsible for a given bend on a given single channel near the playing frequency or at least (following Johnston's model, see chapter

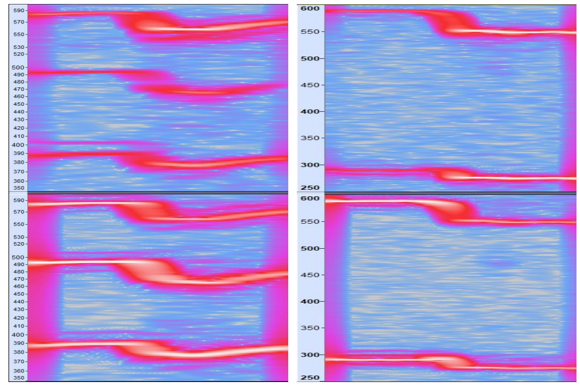

Figure 12: Spectrograms for simultaneously bending a chord (left) and an octave (right), summation signal of oscillations (blow reeds: upper half, draw reeds: lower half of the picture) recorded with help of *ELX*. Left: Bending G4, B4, D5 on channels #2, #3, #4 by 32$cent$, 101$cent$, 73$cent$. Right: Bending D4, D5 on channels #1, #4 by 92$cent$, 115$cent$)

4.4) between the eigenfrequencies of the two reeds in the channel. It seems astonishing that one resonance of the player's oral cavity can simultanesously match two or three different playing frequencies on different channels.

It should be mentioned that, in contrast to my experiments (bending an octave using tongue split), Pat Missin demonstrates on his website [PM] bent and unbent notes played simulataneously. Using a tongue split, he is playing octaves on channels #2 (draw bend) and #5 (normal draw) as well as on channels #3 (draw bend) and #6 (normal draw).

5.4 Slides

Fig. 13 shows the signals of the reed movements detected with the optical sensors of the ELX. Trial 1 starts with a normal draw note D4 on channel #4, which is bent down almost to C4, followed by a rapid slide down to channel #1. During the slide, the geometry of the vocal tract is fixed (according to subjective perception). The spectrogram clearly shows a fairly deep bend on channel #3, a weaker bend on channel #2, and a very soft bend on channel #1. Trial

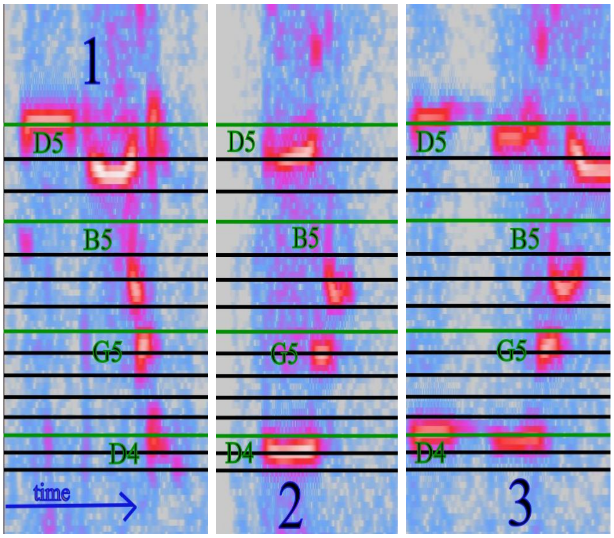

Figure 13: Sliding with draw bends, reed movements recorded with *ELX*. Spectrogram with logarithmic frequency axis. Time axis from left to right. Left: Bending a normal note on channel #4 and sliding down to channel #1 with fixed vocal tract geometry. Middle: Starting with a low bend on channel #4 and sliding upwards. Right: Bending a normal note on channel #1 “weakly” and sliding upwards. The horizontal lines mark half tone steps, in green are highlighted the frequencies of the draw notes on the lower four channels of the C-harp: D4-G5-B5-D5.

2 starts with a strong bend on channel #1, where it can be seen that the reeds also oscillate an octave higher in the first harmonic. Rapid upward sliding produces fairly strong bends on channels #2 and #3, but no sound on channel #4. Finally, trial 3 starts with a soft bend on channel #1. The spectrogram again shows at the same time the first harmonic. A rapid slide upward produces bends on channels #2 and #3 and finally a strong bend on channel #4. In comparison, trial 3 looks like the "inverse" of trial 1.

The slides were played very fast, the notes in the slides sounded for about $0,1s$ with stable pitch[10], with no sound of about $0,02s$ in between, so it seems very unlikely that I should have changed my embouchure from one channel to the next.

Sliding with drawbends again demonstrates that bends with significantly different playing frequencies can be created with one and the same embouchure.

5.5 Bend and overbend

5.5.1 Switching between bend and overbend

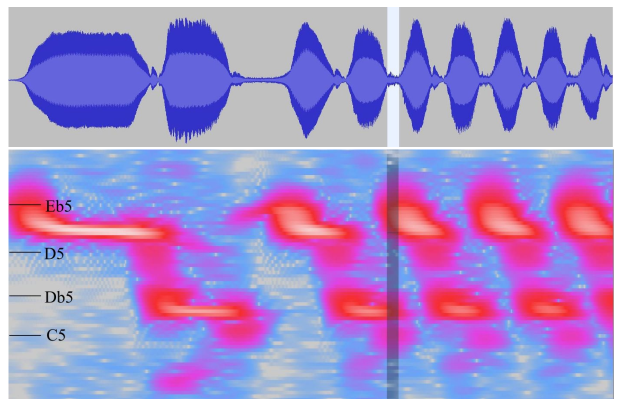

Figure 14: Switching between overblow and draw bend with fixed embouchure, emitted sound recorded with *Zoom H4n* using the built-in microphones. Wave in sync with spectrogram.

The wave diagramm and the corresponing spectrogram in Fig. 14 show overblows and draw bends on channel #4 of a C-harp with frequencies $606Hz$ (overblow) resp. $545Hz$ (draw bend). The series of notes evaluated in the right half of the figure was obtained by rapidly switching between inhaling and exhaling while keeping one’s embouchure fixed. Before and after the marked rectangle one hears clearly a draw bend resp. an overblow of constant playing frequency. The rectangle marks a switching time of ca. $0,07s$. It seems very unlikely that the player’s embouchure has changed significantly during this short time interval.

Thus, as a result, it can be stated that with one and the same geometry of the vowel tract, notes can be played that are approximately one whole tone apart.

[10]One can hear the stable pitch very clearly by changing the velocity of the recording without changing the pitch applying the effects menu of *Audacity*.

It should be mentioned that bend notes played with same embouchure as an overbend seem to be the lowest note in the respective channel, i.e. the bend with lowest feasible frequency (about 30*cent* above the normal blow note on channel #4 of a C-harp).

5.5.2 Bending an overbend

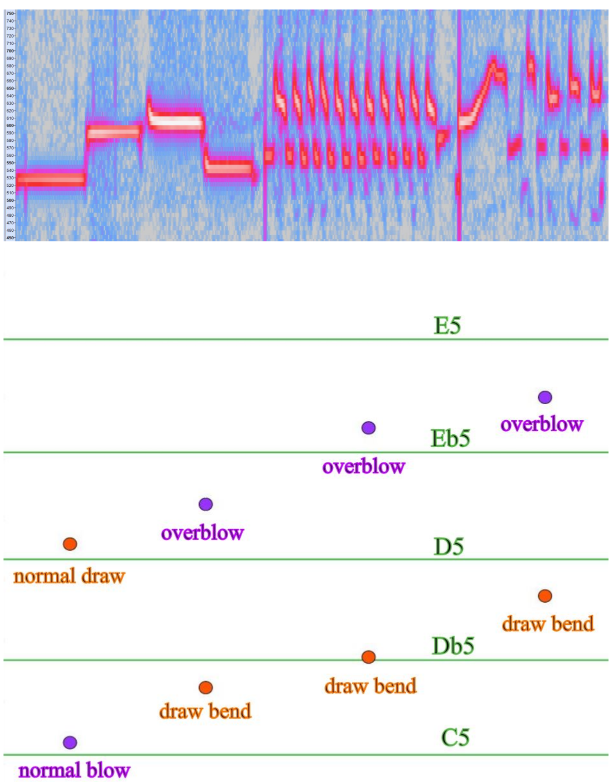

Figure 15: Switching between bended overbend and draw bend, Spectrogram (top) with respective frequencies (bottom) of reed movements recorded with *ELX*.

It is possible to bend overblows upwards. For a bended overblow the corresponding draw bend sounds with higher frequency, so it is less bent (see Fig. 15). Technically, you can switch from an upbent overblow to a weakly bent draw note, but not vice versa. You can only reach the overblow if you start with a maximally bent draw note.

Furthermore, Fig. 15 indicates that the difference between the playing frequencies of a bent overblow and the corresponding drawbend is approximately constant.

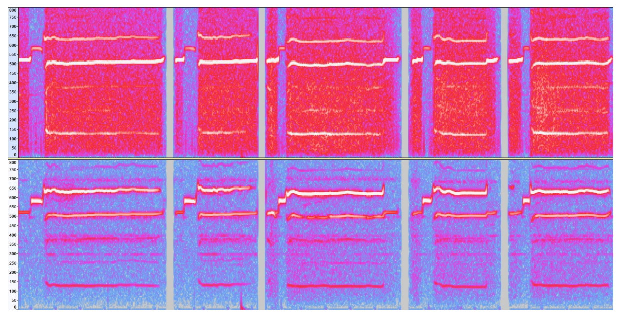

Figure 16: Spectrograms of five trials to play two blow notes simultaneously on channel #4 of a C-harp, reed movements recorded with *ELX*. Upper row: Blow reed, lower row: draw reed.

Blowing into a channel with "overblow embouchure" can result in a sort of "disturbed" overblow, which is appriciated by listeners as a very "rough" mixture of two notes. Fig. 16 shows the spectrograms of five trials to play such a note. The recording was made with an ELX, showing separately the movements of the two reeds (upper row: blow reed, lower row: draw reed). Each trial starts with a short normal blow note, followed by a normal draw. The spectrograms show two dominant frequencies[11], interpretable as an overblow with main contribution coming from the draw reed and as a blow bend with main contribution from the blow reed. The line associated with the overblow is richer in energy, which mirrors the fact that the overblow always "wins " if the player decides to switch from this mixture to a single note. Fig. 17 shows spectrograms of the emitted sound (recorded with microphone, no *ELX*), in which the dominance of the overblow in the sound is clearly seen.

[11] The sound and the spectrogram coming from the blow reed is very "noisy", which might be an artefact.

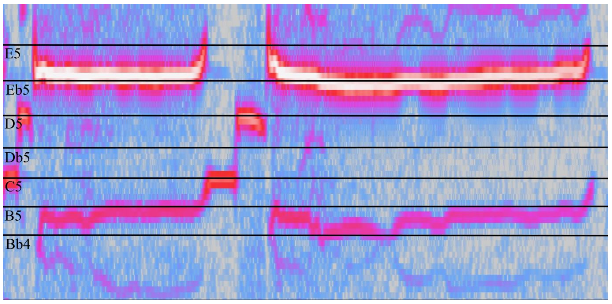

Figure 17: Two trials to play two notes simulteously on channel #4 of a C-harp. Spectrogram of a recording of the emitted sound.

5.6 Switching between harmonicas

The preceding chapters 5.3 and 5.5 contradict a one-to-one relationship between enbouchure and playing frequency. Nevertheless, skilled players claim to be able to "hear" the played note in advance. However, this can also simply be the result of intensive practice. For example, we speak vowels without conscious prior adjustment of the vocal tract. In the same way, my subconscious might "know" how to play a bend note A5 on channel #3 of a C-harp.

The following experiment may at first sight indicate a relationship between embouchure and pitch: Playing bend notes F#4 resp. F4 on channel #2 of a C-harp, fixing one's embouchure (at least according to subjective impression) and switching to channel #3 on an A-harp results in bend notes with *same* pitch F#4 resp. F4. Mind that on channel #2 of a C-harp a G4 was bent down, whereas on channel #3 of an A-harp a different normal note is bent, namely a G#4. Comparing pitches of bend and draw reed, a semitone bend F#4 on the C-harp becomes a whole tone bend F#4 on the A-harp, and a whole tone bend F4 becomes a bend by a minor third on the A-harp. The frequency of the bend note is an invariant.

What, if we change from the C-harp to the Bb-harp? F#4 would be a bend by a minor third on the Bb-harp, but F as a bend by a major third cannot exist. Actually, changing from channel #2 of the C-harp to channel #3 of the Bb-harp ends up in bends with altered pitches near G4 resp. F#4, which amounts to a whole tone bend resp. a bend by a minor third. Now the “distance” to the unbent normal draw note is the invariant, not frequency!

6 Concluding questions

- Why do an opening and a closing reed oscillate after a very short transient period with common frequency, but two closing reeds do not?
- Why does the overblow always win against the blowbend on the lower channels?
- What role do resonance properties of the vocal tract play in determining the playing frequency of bends or overbends?
- How can the continuous transition from normal to bent notes be modeled?
- Can the experiments presented in section 5 be explained within the framework of a resonator model for bends and overbends?

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